UFLOW: Visualizing Uncertainty in Fluid Flow *

Suresh K. Lodha, Alex Pang, Robert E. Sheehan, and Craig M. Wittenbrink

Computer Science Department
University of California, Santa Cruz, CA 95064
{lodha,pang,bob,craig}@cse.ucsc.edu

Abstract

Uncertainty or errors are introduced in fluid flow data as the data is acquired, transformed and rendered. Although researchers are aware of these uncertainties, little has been done to incorporate them in the existing visualization systems for fluid flow. In the absence of integrated presentation of data and its associated uncertainty, the analysis of the visualization is incomplete at best and may lead to inaccurate or incorrect conclusions. This work presents UFLOW – a system for visualizing uncertainty in fluid flow. Although there are several sources of uncertainties in fluid flow data, in this work, we focus on uncertainty arising from the use of different numerical algorithms for computing particle traces in a fluid flow. The techniques that we have employed to visualize uncertainty in fluid flow include uncertainty glyphs, flow envelopes, animations, priority sequences, twirling batons of trace viewpoints, and rakes. These techniques are effective in making the users aware of the effects of different integration methods and their sensitivity especially near critical points in the flow field.

Key Words and Phrases: flow visualization, uncertainty glyphs, streamlines, rakes, flow envelopes, animation.

1 INTRODUCTION

With few exceptions, most of the visualization work done to date have ignored or isolated the presentation of uncertainty from the data. Part of the reason for this practice is the inherent difficulty in defining, characterizing, and controlling the introduction of uncertainty in the visualization pipeline (see Figure 1). Another difficulty is the absence of methods that integrate the presentation of data together with uncertainty. Finally, there is also a need for a framework to evaluate the effectiveness of these visualization methods. This paper focuses on the problem of visually mapping data and uncertainty together for fluid flow.

The practice of presenting data together with uncertainty such as maximum-minimum range or standard deviation is quite common in traditional univariate data display such as Tukey's box plots [27] and Tufte's quartile plots [26]. Unfortunately, this practice of presenting data together with uncertainty information is lost in more recent visualization techniques. Nevertheless, integrated presentation of data with uncertainty is considered a worthy goal in scientific visualization [2, 25]. This paper takes some significant steps towards achieving this goal in fluid flow visualization.

The first step is to characterize the uncertainty in fluid flow. Although researchers have been well aware of these uncertainties [5, 13], no systematic effort has been taken to identify all possible sources of uncertainty and quantify them wherever possible. Only recently have efforts been directed at evaluating and characterizing errors found in visualization software [10]. In Section 2, we shall define uncertainty more formally, and discuss possible sources of uncertainty. In this work, we focus on uncertainty arising due to the use of different numerical algorithms to compute particle traces in fluid flow. In Section 3, we review what has been done to visualize uncertainty.

The second step is to present the fluid flow data together with uncertainty. This is the step that we focus on in this work. What possible visualization techniques can be used to display uncertainty? The number of visual variables available to human eye are limited. Bertin classifies all retinal variables into seven categories – hue, saturation, value, shape, size, orientation, and location [3]. These categories have been utilized to devise clever schemes in computer graphics and scientific visualization. Glyphs and textures are two such examples that convey scientific information about data. In this work, we present several techniques, that span across these categories, expressly to highlight locations and amount of uncertainty found in fluid flow data. These techniques include: uncertainty glyphs, flow envelopes, animations, priority sequences, twirling batons of trace viewpoints, and rakes. Although some of these methods are not necessarily new and have been used in other contexts, they are very useful tools when analyzing the effects of different uncertainty sources on fluid flow streamlines.

2 UNCERTAINTY

2.1 What is Uncertainty?

The NIST standards report on uncertainty [25] and the NCGIA initiative on Visualization of Spatial Data Quality [2] define data uncertainty very broadly to include concepts such as statistical variation or spread, error or inaccuracy, and minimum-maximum ranges. *Statistical* uncertainty is given by the estimated mean and standard deviation, which can be used to calculate a confidence interval, or an actual distribution of the data. *Error* uncertainty is the difference between a known correct datum and an estimate or an absolute valued error among estimates of the data. *Range* uncertainty is an interval in which the data must exist, but which cannot be quantified into either the statistical or error definitions.

2.2 Sources of Uncertainty

In order to understand what is overlooked in visualization, we quickly review the sources of uncertainty, errors, and ranges found in most data sets. Figure 1 illustrates the three major blocks in a visualization pipeline leading to the analysis of the visualization output. It is clear that different forms of uncertainty are introduced into the pipeline as data are acquired, transformed, and visualized. Starting with the *data acquisition* stage, one will note that nearly all data sets, whether from instrument measurements, numerical models, or data entry have measurement uncertainty. With instruments there is an experimental variability whether the measurements are taken by a machine or by a scientist. The more times the measurement is taken, the confidence about the measurement distribution changes. In numerical modeling, the model and its parameters have been decided by a domain specialist, and is inherently a simplification (e.g. linearization of a nonlinear system) of the system being

*http://www.cse.ucsc.edu/research/slvg/uflow.html

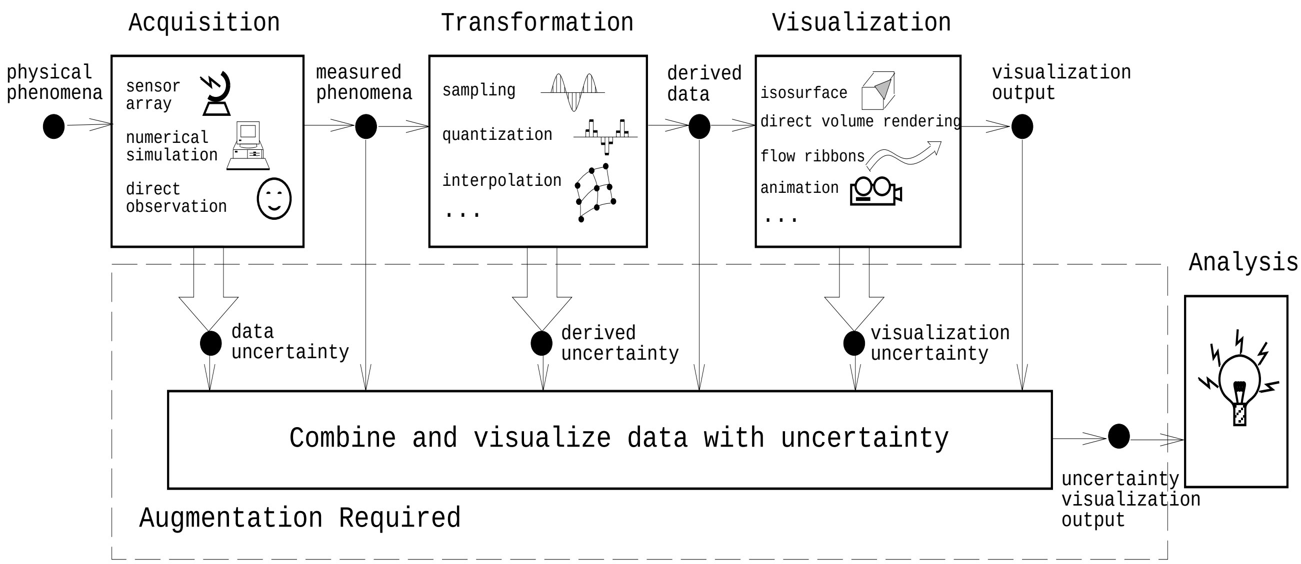

Figure 1: This visualization pipeline shows measurement uncertainty, derived uncertainty, and visualization uncertainty.

modeled. The next important source of uncertainty is in the *data transformation* stage. In addition to model simplification and sensitivity of these models to input parameters, numerical calculations performed on these models also introduce errors due to the integration algorithms and the limited precision of the computing machinery. Finally, uncertainty is also introduced in the *visualization* stage itself due to the errors and approximations associated with the rendering models and algorithms needed in the rendering process. Examples of this include the holes arising from ambiguities in the marching cubes algorithm as suggested originally [16], and the resampling and interpolation errors introduced in fitting scattered data sets [21].

2.3 Uncertainty in Fluid Flow

Features of interests in fluid flow study include position, speed, velocity, acceleration, topology, and critical points [12]. These quantities are not immune from the different sources of uncertainties described in Section 2.2. Many different techniques such as hedgehogs, particle tracing, surface particles, stream polygons, stream ribbons, and streamballs [4] have been used for fluid flow visualization [23]. All these techniques are also subject to the different sources of uncertainties. These uncertainties can however be represented as a scalar or vector valued function that captures the difference, inaccuracy, or confidence level of the flow feature of interest.

For the purpose of this paper, we generate these fluid flow uncertainty values by varying integration methods and parameters used in generating streamlines. The degrees of freedom include (a) selecting between an Euler or 4th order Runge-Kutta integration method, (b) varying the integration time steps of each method, and (c) changing the direction of the integration between forward and backward. For example, two slightly different streamlines can be generated by simply changing the size of integration time steps. Another example is when a particle is traced forward in time, then backward in time from where it ended. We are interested in ways of visually mapping the positional and vector differences between any such pair of streamlines.

It is also important to note that while a suite of integration techniques and options are made available to the user, our goal is exactly to highlight any differences that may arise from the user's choices. In addition, while a higher order integration would in general produce more reliable paths than a first order scheme, the differences may not be significant in most cases unless the path is in the proximity of critical points in the flow field. UFLOW provides the environment for investigating the tradeoff between more expensive methods against data sensitivity. Finally, while our definition of uncertainty encompassed a broad range of concepts, our focus in this paper is on uncertainty arising from simple deterministic differences.

3 EXISTING METHODS OF VISUALIZING UNCERTAINTY IN FLUID FLOW

Many researchers are fully aware of the uncertainty in their data. In flow visualization, these are usually displayed using one of two methods: side-by-side comparison or super-imposition (sandwich layers). For example, Buning [5] and Kenwright [13] use both of these techniques in order to bring out the integration errors. The side-by-side comparison is useful when the differences are large and immediately obvious. Otherwise, it puts too much burden on the user to identify the location and magnitude of the difference. The second method of superimposing works better. However, when comparing streamlines, it is difficult to see if a particle is traveling faster on one path than another.

In other areas of visualization, there are more ideas to draw upon. For example, differencing and pseudo-coloring are two other popular methods. Differencing is most often done on 2D images, but can also be applied to 3D volumes [8]. Pseudo-coloring is then used to indicate the scalar magnitudes in the difference images or volumes. Animation, segmentation, and blurring have also been used to visualize fuzzy data [9]. Yet other techniques for visualizing uncertainty include transparency for comparing surface interpolants [21], haziness corresponding to uncertainty [2], and defocussing or Monte Carlo blurring [7]. Error analysis visualizations are also common in the field of statistical visualization [27, 6, 29].

There are also work on extending visualization techniques such as the use of glyphs and textures to include uncertainty information. Glyphs or icons [22] have often been used for visualizing fluid flows [24]. The traditional arrow vector glyph has recently been extended to encode information about uncertainty in both direction and magnitude [33]. Likewise, texture mapping [11] has been used in scientific visualization [28]. More recently, these techniques, together with iterated function systems have been used to visualize geometric uncertainty of surface interpolants [15, 31, 32], and compare differences of 3D surface attributes [20].

4 UFLOW

UFLOW stands for uncertainty flow visualization. It is not a system for visualizing flow data per se, but rather a system for visualizing the uncertainty found in flow visualization. As such, it is designed to provide users with several means of highlighting these uncertainties. This section will describe the general architecture of UFLOW and the methods available for visualizing flow uncertainties.

4.1 Architecture and Interface

UFLOW is designed around five orthogonal groupings. These groupings allow separate specification of: the source of error, the type of error, the flow visualization method, the visual mapping to be used, and the transfer function to be used. Separation of these parameters into different groupings allow a richer set of combinations to be studied and makes the system more easily extendible.

As illustrated in the graphical user interface in Figure 2, the source of error can be generated by specifying different integration methods and/or a different step size for each method. The type of error can either be positional error or velocity error. Positional components (e.g. x, y, and z) as well as velocity components can also be specified as the type of error to be analyzed. The wide variety of features (e.g. flow patterns, vortices, etc.) and varying needs of researchers give rise to a large collection of flow visualization methods (e.g. ribbons, streaklines, etc.). These methods can potentially suffer from a similar set of error sources and thus need to be analyzed individually. At the moment, UFLOW has the streamlines and rakes method. The visual mappings specify how the error types found in the selected flow method is to be presented to the user. It includes the use of glyphs, envelopes, animation, etc. described in the next section. Transfer functions allow users to select from a set of standard color maps or to create their own in order to bring out the features of interest in their visualization. With these five independent and extendible groupings, one can easily create a rapidly expanding set of uncertainty flow studies.

4.2 Visual Mappings of Data with Uncertainty

We now present six different methods for visually displaying uncertainty in fluid flow visualizations. These methods are uncertainty glyphs, envelopes, animations, ranked animations or priority sequencing, twirling baton display of trace viewpoints, and rakes.

Uncertainty Glyphs:

Glyphs or icons are graphics objects that encode information through their shape, color, size, and other attributes. Uncertainty glyphs are probes which can be placed in a graphic to indicate the confidence interval, error, or range.

In UFLOW, we present users with four different shapes – line segments, balls, bar bells and ellipsoids. Some shapes such as ellipsoids allow three degrees of freedom namely, the radii along the three principal axes, to choose from. These parameters can be mapped independently to three different uncertainty measures or some vector-valued uncertainty. For each of these glyphs, there are several parameters to choose from. Because of the independent grouping described in the previous section, each of these glyphs can be pseudo-colored using a preset color table or a customized transfer function. In addition, the user can scale the size of the glyphs, and change the density, and spacing of the glyphs. These parameters can be adjusted interactively to improve the visualization by reducing clutter, or they can also be mapped to different uncertainty values. For example, the size of the glyph and the color could both be mapped to distance uncertainty using overloading approach or they could be mapped to different uncertainty parameters to visualize correlation between the two parameters.

Envelopes:

Envelopes are strips or tubes that show the range of possible values in the data. They are most appropriate for uncertainty represented by a range of min/max values.

In UFLOW, strips are constructed in a similar fashion to ribbons. That is, two streamlines are bridged together by filling and coloring the space between corresponding particle path positions. On the other hand, tubes are created by using spheres centered at the midpoint of the line joining the two traces with a radius equal to half of the distance between the two traces. The region on the strips, or the space within the tubes indicate the possible positions of a trace starting from the same seed point.

Animations:

This is an alternative way of presenting min/max values. Instead of using envelopes, an uncertainty glyph is animated through the particle trace. This method is effective in attracting user's attention to areas of high uncertainty when the glyphs grow in size or change their color to bright red (using rainbow transfer function for example). A slight variation of animation is *oscillation* when an object is made to oscillate between the possible values.

In UFLOW, all uncertainty glyphs can be animated as the streamlines are traced out. In addition, the user can control the speed of the animation.

Ranked Animations or Priority Sequencing:

Animation of glyphs as the particle traces its path is useful. An even more interesting animation is to rank the animation in some order. The user can specify some criterion to prioritize or rank the points on the traces. The animation is then presented as a sequence in order of this ranking.

UFLOW supports priority sequencing by presenting uncertainty glyphs in order of highest to lowest uncertainty. By presenting this information in this order, the user is immediately alerted to the areas of high inaccuracy. This technique also identifies areas of equal uncertainty that are spatially apart quite well by presenting them immediately one after one another.

Twirling Baton Display of Trace Viewpoints:

This method presents the uncertainty from the viewpoint of a particle traveling down a streamline. Actually, the viewpoint is taken from the midpoint between two streamlines. To understand this, imagine yourself being a particle traveling down the midpoint between two streamlines. As you go down the path, the extremities will change in length and orientation. There are at least two alternative coordinate frames to use. Instead of moving and orienting the coordinate frame along the path trajectory, the local coordinate frame is a simple translation of the world coordinate frame. That is, the frame mapping is achieved by translating the existing coordinate frame to the midpoint between the two streamlines at any given instant. This simplifies calculations and also helps the viewer orient themselves with respect to the world. This view is reminiscent of a twirling baton, hence the name. The longer the baton, higher the uncertainty. More twirling to the baton, more uncertainty in the orientation.

UFLOW displays the twirling baton in a separate window together with an animation of an uncertainty glyph that is being traced along a particle path. This presentation method is particularly useful when the particle traces criss-cross each other a lot, as in a twist-

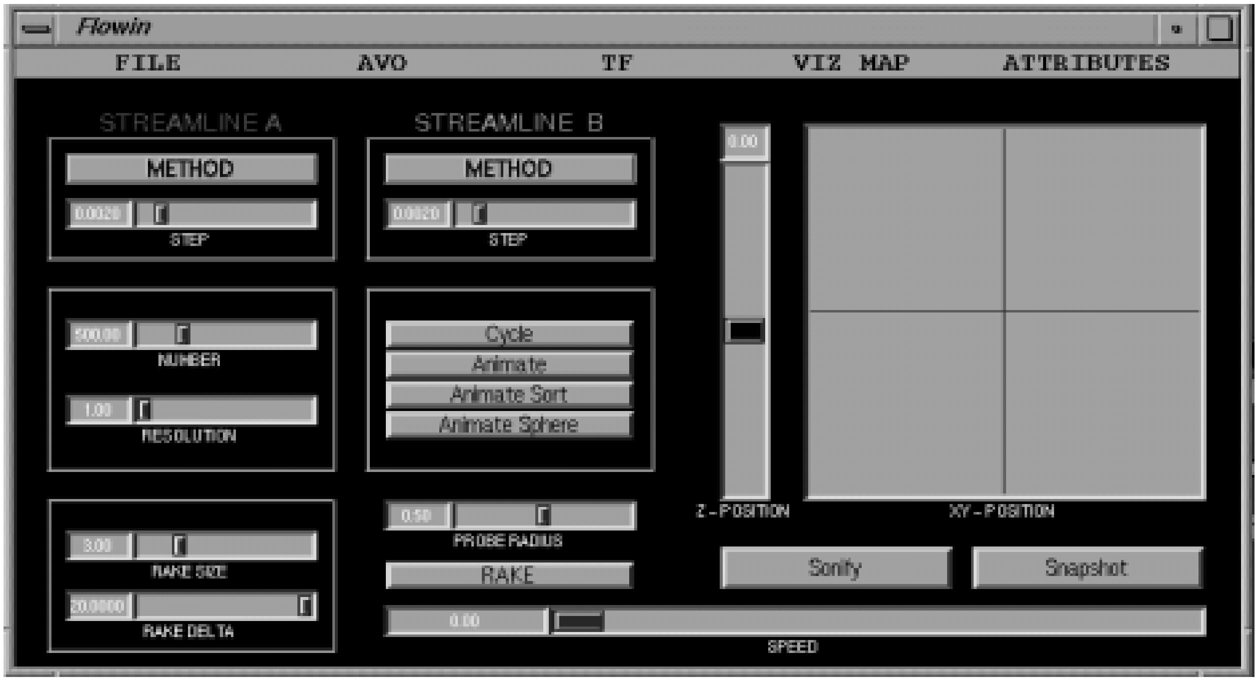

Figure 2: User Interface of UFLOW

ing ribbon.

Rakes:

Finally, rakes are multiple seed points along a line. Rakes have been found to be very useful in exploring fluid flow data particularly in highlighting twists and turns. They can also quickly lead a user to subregions of data including critical points and other interesting topologies.

UFLOW uses rakes as a visualization method and also as a means for visually mapping uncertainty values. In particular, the width and spacing of paths across the rake give a good indication of divergence and convergence properties of the uncertainty values.

5 RESULTS

This section describes the data sets used to test out the visual mapping methods as well as the experiments and results that we obtained.

5.1 Data Sets

We have so far experimented with several analytic data sets described in [13]. These data sets include a helical data flow, a spiral data flow, a sharp corner data flow, a stretch-twist-fold data flow, and a closed loop data flow. Knowing the behavior of these flow data helped us verify the correctness of our visual mapping strategies. We will also be applying these methods to more complex simulated data sets as well. A brief description of the data sets are included below.

The velocity field for the helical data flow is described by: $\mathbf{v}(x, y, z) = -by\mathbf{i} + bx\mathbf{j} + c\mathbf{k}$, where b and c are constants and affect the pitch of the helix. A physical example that produces this pattern is a spinning propeller [30]. Variations of this helical flow are often used for testing streamlines and particle tracking algorithms [19, 18, 17, 13].

The 3-d mass conservative velocity field for the spiral data flow is given as: $\mathbf{v}(x, y, z) = (ax - by)\mathbf{i} + (bx + ay)\mathbf{j} + (-2az + c)\mathbf{k}$. This spiral flow pattern is similar to an idealized vortex such as from the tip of an airplane wing.

The sharp corner fluid flow data is described by the following velocity field: $\mathbf{v}(x, y, z) = x(x - y)\mathbf{i} + y(y - x)\mathbf{j} - z(x + y)\mathbf{k}$. This flow field exhibits a more complex figure eight flow pattern.

In the closed loop flow, flows are confined to move in a closed region. That is, streamlines return to the point where they started from, thereby forming a closed loop. The velocity field used to generate this fluid flow is: $\mathbf{v}(x, y, z) = 8(8yz - 8xy - 7z + 11x - 6)\mathbf{i} + 64(z^2 - x^2 + 3x - 3z)\mathbf{j} + 8(8yz - 8xy + 7x - 11z + 6)\mathbf{k}$.

In real circulating flows, it is more likely that the streamlines pass near the starting points rather than return exactly to their starting points. For example, Bajer *et al.* [1] describes such a flow, which is an incompressible three-dimensional stretch-twist-fold fluid flow. The velocity field is given by: $\mathbf{v}(x, y, z) = (\alpha z - 8xy)\mathbf{i} + (11x^2 + 3y^2 + z^2 + \beta xz - 3)\mathbf{j} + (-\alpha x + 2yz - \beta xy)\mathbf{k}$, where $\alpha > 0$ produces streamlines that spread to fill the entire domain.

5.2 Examples

Using the color plates, we describe six examples of how differences in streamlines can be visually displayed in UFLOW.

Color plate 1 uses closely spaced sphere glyphs positioned midway between particle positions on two streamlines. The effect is a tube that indicates the envelope of where the streamline might lie. A hot-to-cold color table and the size of the spheres are used to map the magnitude of deviation between the two streamlines.

On the same data set, Color plate 2 uses colored strips to tile between the two diverging streamlines. The appearance is similar to ribbons used in flow visualization. The values being mapped to the same hot-to-cold color table is the x component of the particle velocity. Unlike the previous example, the difference between the two streamlines is not monotonically increasing.

Color plate 3 uses a similar strategy to tiling colored strips. Here, line glyphs are used to span across the two streamlines. Again color is used to indicate magnitude of difference. A monotonically increasing divergence from the initial seed point can be observed for the helical data set.

In contrast to the other illustrations, Color plate 4 shows the differences between the two streamlines from the point of view of a particle traveling down halfway between the two. This is a snapshot of an animation sequence that shows how the barbell (two spheres, indicating the instantaneous particle positions on the two streamlines, connected by a line segment) changes in length and orientation corresponding to magnitude and twist of the two streamlines. A sliding set of the last ten barbells are kept in the display in order to give an indication of the immediate past history of the trajectory.

Color plate 5 uses the rake visualization method to highlight divergence of multiple streamlines. Here, multiple closely spaced but adjustable seed points are used to trace out streamlines. Barbells are then used to show the difference between each pair in the rake. This method is very useful for studying effects near critical points.

Color plate 6 shows barbells in more detail. The line segments joining the spheres draws the user to the unequal rate of particle advancement along their paths. Barbell glyphs illustrate shearing in streamline pair.

6 CONCLUSIONS AND FUTURE WORK

In this paper, we presented several methods for visualizing uncertainty in streamlines of flow data arising from different integration algorithms. These methods include uncertainty glyphs, envelopes, animations, priority sequencing, trace viewpoints, and rakes. We have tested these methods on analytical data sets. The resulting visualizations of data and uncertainty are integrated together. This is an important first step towards presenting an accurate depiction of the data to the user. Recently UFLOW has also been extended to incorporate sonification [14]. We believe that these methods will prove valuable to people who need to make informed decisions based on imperfect data or processes.

We plan to extend this work in several directions. First, UFLOW can be easily extended to incorporate new flow visualization algorithms aside from streamlines. Second, UFLOW will be tested on other flow data sets including both simulated and experimental data sets. Third, UFLOW will be evaluated by assessing the performance of users on certain well-defined tasks on this system. Finally, although we have demonstrated the utility of the visualization techniques for displaying uncertainty in particle traces, we believe that we will have to invent additional visualization techniques for displaying uncertainty associated with other abstract visualization objects such as stream ribbons, streamballs and stream tubes used in fluid flow visualization.

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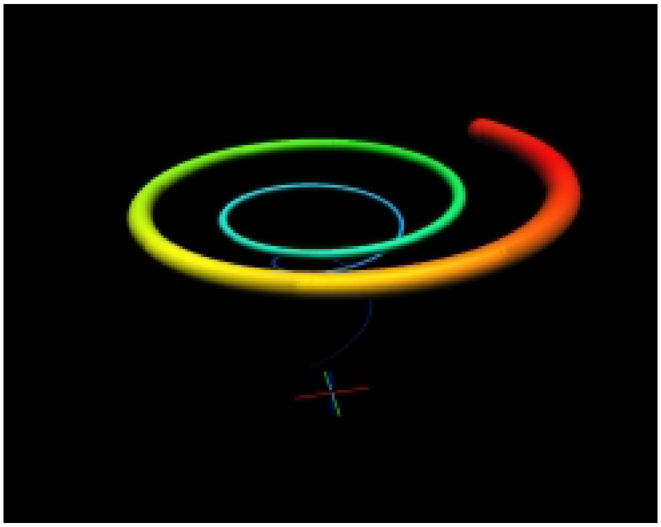
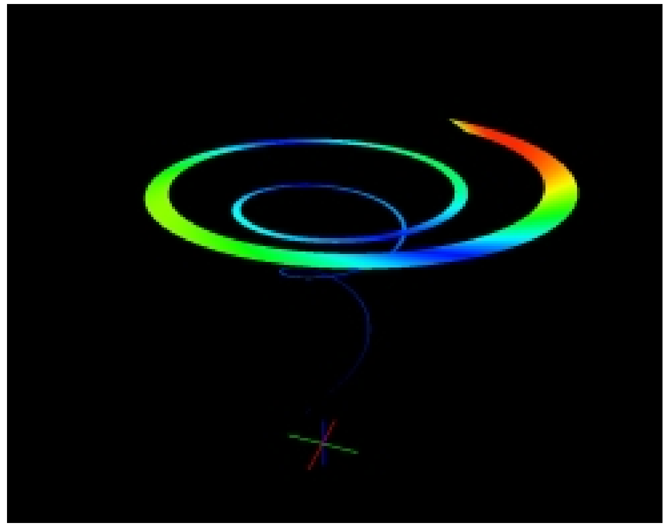
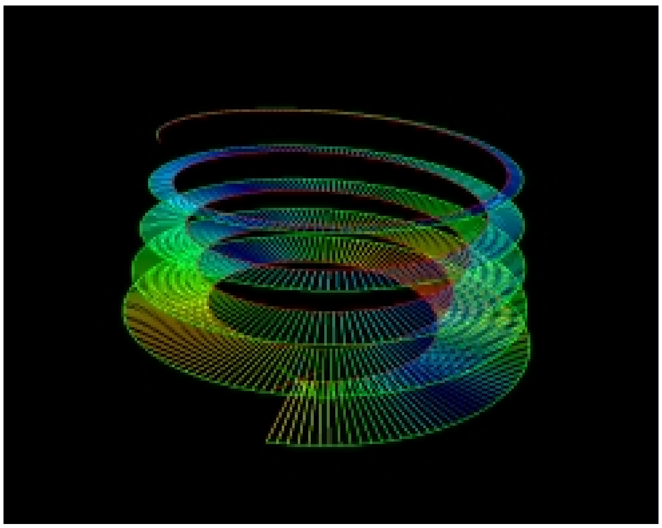


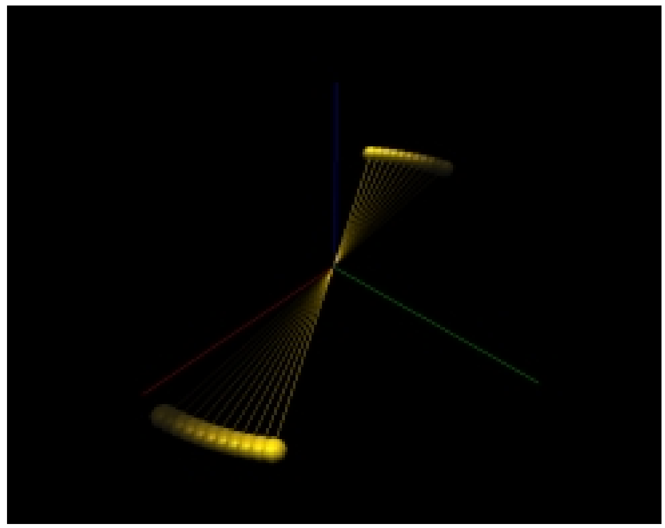
Color Plate 1: Envelope of path trajectory.



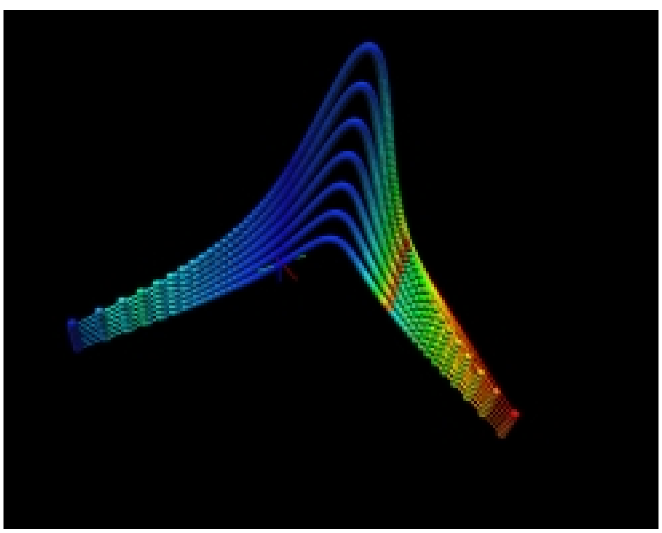
Color Plate 2: Uncertainty ribbon.



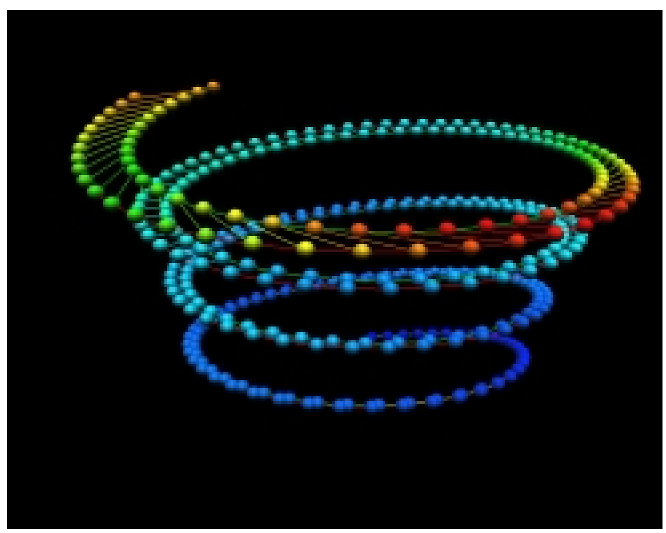
Color Plate 3: Line segment glyphs.



Color Plate 4: Twirling baton.



Color Plate 5: Divergence rake.



Color Plate 6: Shearing barbell glyphs.