Final Exam

CS 4810/6810 – Artificial Intelligence

Due Dec 11, 2009

Please remember to put your name on your answer sheet.

The exam is open book & notes but NOT, "open neighbor." This is an individual exam. Do not work in groups or consult with others.

Write your answers in the space below the questions. If you need additional room, continue on the back of the page, and *be sure to draw an arrow to indicate that there is more on the back.* Extra paper or MS Word or another program can be used. If you use extra paper, make sure to put your name at the top of each new page, and include the extra pages with your exam when you turn it in. The exam should be emailed!

Good luck, and have a very nice holiday!

Rules, Equations and Special Notes.

Inference Rules for Propositional Logic

Modus Ponens (a.k.a. Implication-Elimination)

$\alpha \Rightarrow \beta, \quad \alpha \quad \vdash \quad \beta$

And-Elimination

$\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n \quad \vdash \quad \alpha_i$

And-Introduction

$\alpha_1, \alpha_2, \ \ldots, \alpha_n \quad \vdash \quad \alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n$

Or-Introduction

$\alpha_1 \quad \vdash \quad \alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n$

Double-Negation Elimination

$\neg\neg\,\alpha \quad \vdash \quad \alpha$

Unit Resolution

$\alpha \vee \beta, \quad \neg\beta \quad \vdash \quad \alpha$

Resolution

$\alpha \vee \beta, \quad \neg\beta \vee \gamma \quad \vdash \quad \alpha \vee \gamma$

Additional Inference Rules for First-Order Logic

Universal Elimination

$$\frac{\forall v \quad \alpha}{SUBST(\{v/g\}, \alpha)}$$

Existential Elimination

$$\frac{\exists v \quad \alpha}{SUBST(\{v/k\}, \alpha)}$$

Existential Introduction

$$\frac{\alpha}{\exists v \; SUBST(\{g/v\}, \alpha)}$$

Standard Logical Equivalences

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	de Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	de Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Generalized Modus Ponens

$$\frac{p_1', p_2', \cdots, p_n', (p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q)}{SUBST(\theta, q)}$$

Generalized Resolution

$$\frac{p_1 \vee \cdots p_j \cdots \vee p_m, q_1 \vee \cdots q_k \cdots \vee q_n}{SUBST(\theta,\ p_1 \vee \cdots \vee p_m \vee q_1 \vee \cdots \vee q_n \mathit{except}\ p_j \mathit{and}\ q_k)}$$

Unit Resolution

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k}$$

Full Resolution

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

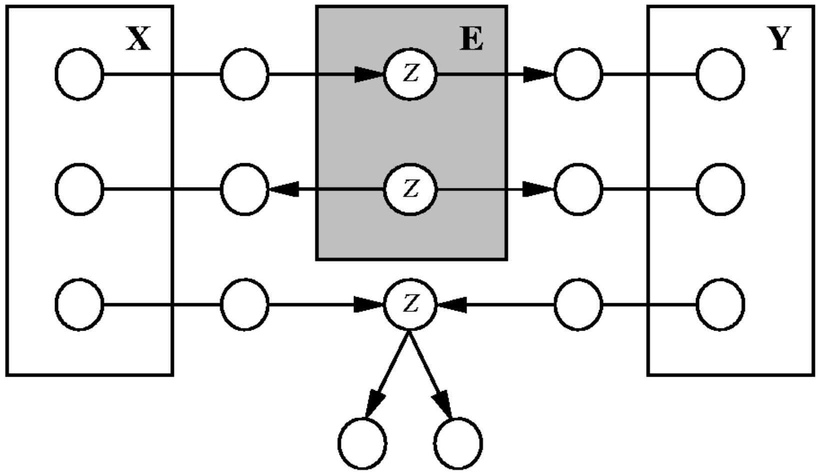
Procedure to convert sentences to Conjunctive Normal Form:

- Eliminate equivalence: Replace $P \Leftrightarrow Q$ with $(P \Rightarrow Q)$ and $(Q \Rightarrow P)$
- Eliminate implications: Replace $(P \Rightarrow Q)$ with $(\neg P \vee Q)$
- Move $\neg$ inwards: $\neg\forall$, $\neg\exists$, $\neg\neg$, $\neg(P \vee Q)$, $\neg(P \wedge Q)$
- Standardize variables apart: $\forall x\ P \vee \exists x\ Q$ becomes $\forall x_1\ P \vee \exists x_2\ Q$
- Move quantifiers left in order: $\forall x\ P \vee \exists y\ Q$ becomes $\forall x\ \exists y\ P \vee Q$
- Eliminate $\exists$ by "Skolemization" – when $\exists$ is on the outside, do existential elimination; otherwise use a "skolem function" $H(x_i)$ to enclose the universally quantified variables
- Drop universal quantifiers
- Distribute $\wedge$ over $\vee$, e.g.: $(P \wedge Q) \vee R$ becomes $(P \vee R) \wedge (Q \vee R)$
- Flatten nesting: $(P \wedge Q) \wedge R$ becomes $P \wedge Q \wedge R$

Procedure to convert FOL sentences to Conjunctive Normal Form:

- Eliminate equivalence: Replace $P \Leftrightarrow Q$ with $(P \Rightarrow Q)$ and $(Q \Rightarrow P)$
- Eliminate implications: Replace $(P \Rightarrow Q)$ with $(\neg P \vee Q)$
- Move $\neg$ inwards: $\neg\forall$, $\neg\exists$, $\neg\neg$, $\neg(P \vee Q)$, $\neg(P \wedge Q)$
- Standardize variables apart: $\forall x\ P \vee \exists x\ Q$ becomes $\forall x_1\ P \vee \exists x_2\ Q$
- Move quantifiers left in order: $\forall x\ P \vee \exists y\ Q$ becomes $\forall x\ \exists y\ P \vee Q$
- Eliminate $\exists$ by "Skolemization" – when $\exists$ is on the outside, do existential elimination; otherwise use a "skolem function" $H(x_i)$ to enclose the universally quantified variables
- Drop universal quantifiers
- Distribute $\wedge$ over $\vee$, e.g.: $(P \wedge Q) \vee R$ becomes $(P \vee R) \wedge (Q \vee R)$
- Flatten nesting: $(P \wedge Q) \wedge R$ becomes $P \wedge Q \wedge R$

***d*-separation:**



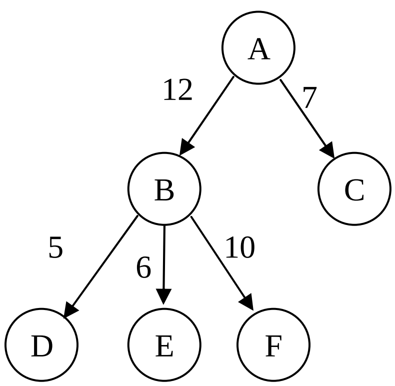
Bayes rule: $$P(H \mid D) = \frac{P(D \mid H)\, P(H)}{P(D)}$$

Conditional probability: $$P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$

Marginalization: $$P(X) = \sum_{Y} P(X,Y) = \sum_{i=1}^{n} P(X, Y = y_i)$$

1. [2 points] What are the three parts of a typical AI program? Briefly describe each.

2. [3 points] The graph below shows an A* search in progress. Which node should be expanded next: C, D, E, or F? Why? (The numbers on the graph are path costs from one node to the next; the table on the right gives estimated distance to the goal, the admissible heuristic used.)



Node	Estimated distance to goal
A	22
B	11
C	16
D	4
E	4
F	2

3. [3 points] Give an example of (a) a "Weak AI" system and (b) a "Strong AI" system. (This doesn't have to be a historical or commercial system, such as Deep Blue; for example "a chess-playing machine" might be one of the examples. But don't use chess or any other game as your examples.)

4. [3 points] Let's say you have some inference procedure *i* that can derive the sentence "P" and also the sentence "$\neg P \vee Q$". What else, if anything, are you <u>certain</u> the inference procedure *i* can derive?

5. [2 points] A knowledge base contains statements about CSUEB departments – variables in this domain can only refer to department names (ComputerScience, Geography, Psychology, ElectricalAndComputerEngineering, …) and a few adjectives (Great, Rotten, Easy, Hard). For the sentence:

$$\forall \mathbf{x}\ \mathbf{IsMajor(x)} \wedge \neg\mathbf{Equals(x, ComputerScience)} \Rightarrow \mathbf{MajorProperty(x, Easy)}$$

Use Universal Elimination to rewrite the sentence.

6. [2 points] What does it mean for an inference procedure to be *sound*?

7. [3 points] Convert the following sentences to Conjunctive Normal Form

   a) $\neg\exists w\, \neg(P(w) \Rightarrow Q(w))$

   b) $\forall y\, \neg Q(y) \vee S(y)$

   c) $\exists y\, \forall x\, P(x) \vee R(x, y)$

8. [2 points] What is an *expert system*?

9. [3 points] Give an English interpretation for the following FOL sentences, making reasonable assumptions about the semantics:

   a) $\exists\, x\ \forall\, y$ IsGreaterThan(x, y)

   b) $\forall\, x, y$ Teammates(x, y) $\Leftrightarrow \exists\, z$ Team(z) $\wedge$ Member(x, z) $\wedge$ Member(y, z)

   c) $\forall\, x, y$ Teammates(x, y) $\Rightarrow \exists\, z$ Team(z) $\wedge$ Member(x, z) $\wedge$ Member(y, z)

10. [3 points] What is the unifier θ of the following pairs of sentences, such that SUBST(θ, p) = SUBST(θ, q)? If the sentences cannot be unified, state so.

    a) p = Symptom(*Flu*, *Fever*) q = Symptom(x, Fever)

    b) p = Symptom(*Flu*, *Fever*) q = Symptom(x, x)

    c) p = Symptom(*Flu*, x) q = Symptom(y, MostCommonSymptom(y))

11. [5 points] On the back, draw a belief net (Bay esian network) for the following scenario, showing all relevant nodes and arcs. Use short (one or two letter) names for the variables and state what they stand for (e.g., “BS” – believes in Santa).

    Christmas presents (a lot, a few, a lump of coal) depend on one’s behavior during the year (very good, average, bad) and whether or not one believes in Santa (believe, don’t believe). Behavior during the year also affects how many friends one has (many, not many). A person’s level of happiness (happy, not happy) depends on how many friends one has and how much fun one has (lots, little), which depends on how many presents one receives at Christmas.

    a) What is the joint probability function for this network?

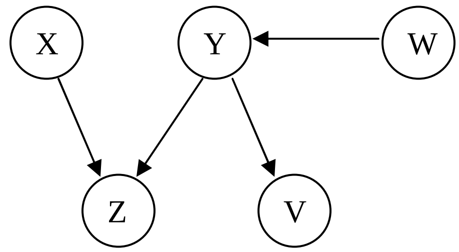
    b) If you are given the CPTs for this network, show a way to compute P(Behavior = bad, Santa = believe).

    c) List all pairs of variables that are independent.

    d) List all pairs of variables that are conditionally independent given Fun.

    e) Show how you would calculate the probability that someone is not happy. (Don’t do the calculation, just set it up.)

12. [3 points] For the following influence diagram, answer the following questions with YES or NO and a very brief explanation.



a) Are W and V independent?

b) Are X and V independent given Y?

c) Are X and Z independent of W, given V?

13. [1 points] Why is probabilistic reasoning often preferable to formal logic in real-world applications?