

CS 413: Analysis of Algorithms
Homework 2

Instructor: Zahra Derakhshandeh

Due **before** the *beginning of your class*

(11:00 am) on Monday, 10/01/2018, for 20189_CS_413_04_1

(2:00 pm) on Monday, 10/01/2018, for 20189_CS_413_03_1

This is an *individual* homework assignment. All solutions has to be **typed**. No credit will be received for hand written solutions (except for figures or long equations which might be handwritten). You need to submit your assignment in the digital repository **before the beginning of your class on Monday, Oct. 1, 2018**.

As it appears in the course syllabus, “for the homework assignments students are *encouraged to discuss* the problems with others, but one is expected to turn in the results of one’s own effort (not the results of a friend’s efforts)”. Even when not explicitly asked you are supposed to concisely justify your answers.

1. For each of the following recurrences, solve the recurrence relation using the Master Theorem and give an expression for the runtime $T(n)$ accordingly. Justify your answer.

(1) $T(n) = 4T(n/2) + n^2$

(2) $T(n) = T(n/2) + 2^n$

(3) $T(n) = 16T(n/4) + n$

2. Consider a complete binary tree T of n nodes, where $n = 2^d - 1$ for some $d \geq 1$. Assume that each node $v \in T$ is labeled with a real number r_v . You may assume that node labels are all distinct. A node $v \in T$ is a local minimum if the label r_v is less than the label r_y for all nodes y that are joined to v by an edge. You are given such a binary tree T but the labeling is only specified in the following implicit way: for each node v you can determine the value r_v by probing the node

- v . Show how to find a local minimum of T using only $O(\log n)$ probes to the nodes of T .
3. Suppose now that you're given an $n \times n$ grid graph G . (An $n \times n$ grid graph is just the adjacency graph of an $n \times n$ chessboard. To be completely precise, it is a graph whose node set is the set of all ordered pairs of natural numbers (i, j) , where $1 \leq i \leq n$ and $1 \leq j \leq n$; the nodes (i, j) and (k, l) are joined by an edge if and only if $|i-k|+|j-l| = 1$.) We use some of the terminology of the previous question. Again, node v is labeled by a real number r_v ; you may assume that all these labels are distinct. Show how to find a local minimum of G using only $O(n)$ probes to the nodes of G , where G has n^2 nodes. What is the run time of your algorithm?
 4. Explain how the divide-and-conquer algorithmic technique works in general? Specify the steps of the divide-and-conquer algorithm for quick sort algorithm (First present the quick sort algorithm and then explain the steps accordingly). In which situation we are able to have $O(n \log n)$ as the running time of this algorithm. Prove your claim.